Final Exam topics
Things to know for **Final Exam, Monday July 27th or Tuesday July 28th**

1.1 Backgroud/Terminology

Differential Equation, ODE, PDE, dependent variable, independent variable, order, linear ODE, nonlinear ODE
ex p. 5-6 #1, 5,7, 13, 15,16

1.2 Solutions to Diff.Eq.'s

- What does it mean to be a solution?
- Explicit vs. implicit solutions
- Use the Existence and Uniqueness Theorem
  Problem Session 3 # 1,2
  ex. p. 14 #23, 25, 27
- Show if a given function is or is not a solution to an IVP (Initial Value Problem).
  ex p. 13-14 #3, 5, 9, 11

2.2 Separable Equations ($\frac{dy}{dx} = f(x)g(y)$)

- Recognize when a Diff.Eq. is seperable
- Find the general solution
- Solve an Initial Value Problem

Problem Session 1 # 5,6
ex p. 46 # 7, 9, 11,17, 23, 24, 25, 29, 34, 36

2.3 First Order Linear ($a_1(x)\frac{dy}{dx} + a_0(x)y = b(x)$)

- Find the general solution
- Solve an Initial Value Problem

Problem Session 1 # 7
Problem Session 3 # 5
ex p. 54 # 7,13,17,21

2.4 Exact Equations

- Test for Exactness
- Find the implicit solution ($F(x, y) = C$)

Problem Session 2 # 2
ex p. 64 #9,11,17,21,22, 29

3.2 Mixing Problems

- Take words, formulate an Initial Value Problem
- Formula: $\frac{dy}{dt} = F_iC_i - F_oC_o$
- Be able to handle cases when $F_i = F_o$ and when $F_i \neq F_o$.

Problem Session 3 # 3
ex. p. 100 #1,3,4,8

4.2 & 4.3: 2nd Order Linear Homogeneous Diff.Eq.

- Find the auxiliary equation
- Find the general solution (3 cases)

- Solve an IVP

Problem Session 2 # 3,4
ex p. 164 #3,7,13,15,18, 26,37,43
ex p. 172 #1,3,19, 21,25

4.4 & 4.5: Undetermined Coefficients & Superposition Principle

- Find y_c
- Use the method of Undetermined Coefficients to find y_p
- Know what to do if y_c and y_p overlap
- Solve IVP's

Problem Session 4 # 1,2
Problem Session 6 # 1 ex p. 180 #13,15,21,33,34
ex p. 185 #1,17,19,21,23,25,33,35

4.6: Variation of Parameters

- Know when to use Variation of Parameters
- Use the method of Variation of Parameters to find y_p
- Solve IVP's

Problem Session 4 # 3,4 ex p. 191 #1,2,3,5,7,13,15

4.9 & 4.10: Springs

- Know how to setup the spring equation $mx'' + bx' + kx = F_{ext}$
- Solve IVP's if $F_{ext} = 0$ and $F_{ext} \neq 0$.
- Know different kinds of damping
- Be able to find a steady-state solution

Problem Session 4 # 5
Problem Session 6 # 2
ex p. 220 # 1,7,9 (You will NOT have to find amplitude, period and frequency, etc.)
ex p. 228 # 9,11,13

7.2 & 7.3: Laplace Transforms

- Know the definition of Laplace Transform, and be able to use it
- Find the domain of the Laplace Transform
- Find the Laplace Transform of a function from a table
- Know the properties:

$$\begin{aligned}
L\{c_1\ f + c_2\ g\} &= c_1\ L\{f(t)\} + c_2\ L\{g(t)\} \\
L\{e^{at} f(t)\} &= L\{f(t)\} \text{ as a function of } (s-a) \\
L\{t^n f(t)\} &= (-1)^n\ \tfrac{d^n}{ds^n} L\{f(t)\} \\
L\{f'(t)\} &= sL\{f(t)\} - f(0) \\
L\{f''(t)\} &= s^2 L\{f(t)\} - sf(0) - f'(0) \\
L\{u(t-a)f(t-a)\} &= e^{-as} L\{f(t)\} \\
L\{u(t-a)f(t)\} &= e^{-as} L\{f(t+a)\} \\
L^{-1}\{e^{-as} F(s)\} &= f(t-a)u(t-a)
\end{aligned}$$

Problem Session 4 # 6a
Problem Session 6 # 3
ex p. 360 #1,3,9,12
ex p. 356 #13,15,17,19
ex p. 365 #1,3,5,9,21

7.4: Inverse Laplace Transforms

- Know the method of partial fractions

Problem Session 5 # 1,2,5
Problem Session 6 # 5
ex p. 374-375 # 1,3,5,21,23,25

7.5: IVP's with Laplace Transforms

- Know the method of Laplace Transforms

Problem Session 5 # 3
Problem Session 6 # 6
ex p. 382-383 #1,3,4,5,11,25,35

7.6: Unit Step Functions

- Know the definition of the unit step function
- Express a function using unit step functions and be able to compute its Laplace Transform
- Express an Initial Value Problem using unit step functions and be able to solve the IVP

Problem Session 5 # 4,6
Problem Session 6 # 4
ex p. 390 #5,7,11,13,15,19,21,23,29,33

9.1-9.3: Matrices

- Know basic operations (multiplication, addition, etc.)
- Find the determinant of 2×2's and 3×3's
- What does it mean if the determinant is zero?
- Find the inverse of 2×2's and 3×3's
- Understand what it means for a Matrix to be a solution to a system of Differential Equations

Matrix Inverse Handout
Problem Session 9 # 4
ex. p. 513 #1,4,9,11,13,17,21,25,37

9.4: Linear systems in Normal Form

- Know how to write systems of equations in Matrix Form (i.e. $\vec{x}' = A\vec{x} + \vec{f}$)

ex. p. 521 #1-11 odd

9.5: Homogeneous Linear Systems with Constant Coefficients

- Know the definition of an eigenvalue and eigenvector
- Given a matrix find its eigenvalues and eigenvectors
- Find the general solution to $\vec{x}' = A\vec{x}$
- Solve IVP's

Problem Session 7 #1-4
ex. p. 531-532 #1,5,16,31,34

9.6: Complex Eigenvalues

- Know the form of the general solution if we have complex eigenvalues
- Find the general solution to $\vec{x}' = A\vec{x}$
- Solve IVP's

Problem Session 7 #5-7
ex. p. 537 #1,3,13

9.7: Nonhomogeneous Linear Systems

- Know the method of Undetermined Coefficients
- Know the method of Variation of Parameters
  (**Important Equation**: $\vec{x}_p = X \int X^{-1} \vec{f}\, dt$)

(You'll be told which one to use on the test)
Problem Session 8 #1-3
Problem Session 9 # 1-3
ex. p. 542-543 #1,3,7,11,13,15,21a

5.1 & 5.2: Interconnected Tanks

- Be able to set up an interconnected tank problem

Problem Session 8 #4-5
ex. p. 250 #31 and p. 544 #34

5.6 Coupled Mass Springs

- Be able to set up the differential equations that describe the system

Problem Session 10 #1a&b
ex. p. 289 #1, 3

5.4 Phase Plane

- It is likely that you will have to do simple graphs on your final.
- Be able to solve the phase plane equation

Problem Session 10 #2,3,4
ex. p. 272-273 #1,3,5,11,13

12.2 Linear Systems in the plane

- Memorize chart on p. 732
- Classify critical points both at origin and not

Problem Session 10 #5,6,7
ex. p. 733 #1-11 odd

12.3 Almost Linear

- Classify critical points (at the origin and when there is more than one critical point)
- Show a system is almost linear

Problem Session 10 #8,9
ex. p. 743 #1,3,9,11