Problem Session 2 (5/28)

1. A man was found dead in Alaska at an unknown time. At 6:00 pm the forensic specialist arrived and found the man's body temperature to be $30°F$. At 6:05 the body's temperature was $28°F$. This led the coroner to deduce that for the body, $k = .75$.

   The temperature outside at 6:00 pm is $25°F$, and will decrease tonight. The daily temperature cycle repeats every 24 hours, and varies by 20 degrees above and 20 degrees below $25°F$.

   Assume the man's body temperature was $98.6°F$ when he was killed. What time was the man killed?

2. Solve the initial value problem

$$(2xy + 3x^2)dx + (1 + 2y + x^2)dy = 0 \qquad y(1) = 2$$

3. Find the particular solution to the Differential Equation $2y'' + 5y' + 3y = 0$ so that it satisfies $y(0) = 3$ and $y'(0) = -4$.

4. Find the particular solution to the Differential Equation $y'' - 2y' + 2y = 0$ so that it satisfies $y(0) = 1$ and $y'(0) = 2$.

5. Find the general solution to $y''' + 2y'' + 5y' = 0$.

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- Newton's Law of Heating / Cooling

$$\frac{dT}{dt} = K(E - T)$$

$T(t)$ = temp. of body

t = measured in hours

$E(t)$ = temp. of environment

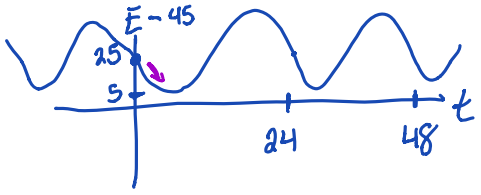
- Stuff we Know

$T(0) = 30°F$

$t = 0 \leftrightarrow$ 6:00 pm

$K = 0.75$

- Find $E(t)$



$$E(t) = 25 - 20\sin\left(\frac{\pi}{12}t\right)$$

$\frac{\pi}{12}$: period of 24 hours

- Find $T(t)$

$$\frac{dT}{dt} = .75\left(25 - 20\sin\left(\frac{\pi}{12}t\right) - T\right)$$

$$= .75(25) - .75(20)\sin\left(\frac{\pi}{12}t\right) - .75T$$

*First order Linear Diff. Eq.

Standard Form: $\frac{dT}{dt} + .75T = 18.75 - 15\sin\left(\frac{\pi}{12}t\right)$

$$\mu(t) = e^{\int .75\,dt} = e^{.75t}$$

multiply: $e^{.75t}\frac{dT}{dt} + e^{.75}(.75)T = 18.75e^{.75t} - 15e^{.75t}\sin\left(\frac{\pi}{12}t\right)$

simplify: $\frac{d}{dt}\left[e^{.75t}T\right] = 18.75e^{.75t} - 15e^{.75t}\sin\left(\frac{\pi}{12}t\right)$

integrate:

$$e^{.75t}T = \int 18.75e^{.75t} - 15e^{.75t}\sin\left(\frac{\pi}{12}t\right)\,dt$$

$$= \frac{18.75}{.75}e^{.75t} - \int 15e^{.75t}\sin\left(\frac{\pi}{12}t\right)\,dt$$

integration by parts twice

$$e^{.75t}T = 25e^{.75t} - \left(\frac{1}{1-\frac{\pi}{81}}\right)\left(20e^{.75t}\left(\sin\left(\frac{\pi}{12}t\right) + \frac{\pi}{9}\cos\left(\frac{\pi}{12}t\right)\right)\right) + C$$

$$T = 25 - \frac{20}{1-\frac{\pi}{81}}\left(\sin\left(\frac{\pi}{12}t\right) + \frac{\pi}{9}\cos\left(\frac{\pi}{12}t\right)\right) + \frac{C}{e^{.75t}}$$

- Use initial condition $T(0) = 30$

$$T(0) = 30 = 25 - \frac{20}{1-\frac{\pi}{81}}\left(\cancel{\sin(0)} + \frac{\pi}{9}\underbrace{\cos(0)}_{1}\right) + \frac{C}{1}$$

$$5 = \frac{20}{1-\frac{\pi}{81}}\left(\frac{\pi}{9}\right) + C$$

$$5 - \frac{20}{1-\frac{\pi}{81}}\left(\frac{\pi}{9}\right) = C$$

$$\approx 12.263$$

$$T(t) = 25 - \frac{20}{1-\frac{\pi}{81}}\left(\sin\left(\frac{\pi}{12}t\right) + \frac{\pi}{9}\cos\left(\frac{\pi}{12}t\right)\right) + \frac{\left(5 - \frac{20}{1-\frac{\pi}{81}}\left(\frac{\pi}{9}\right)\right)}{e^{.75t}}$$

- Find t when $T = 98.6$

$$98.6 = 25 - \frac{20}{1-\frac{\pi}{81}}\left(\sin\left(\frac{\pi}{12}t\right) + \frac{\pi}{9}\cos\left(\frac{\pi}{12}t\right)\right) + \frac{\left(5 - \frac{20}{1-\frac{\pi}{81}}\left(\frac{\pi}{9}\right)\right)}{e^{.75t}}$$

solve for t

on a test, I'll give you an easy equation

on homework, use WolframAlpha, Symbolab, Maple, etc

$t \approx -2.2819\ldots$ $\leftrightarrow$ **3:43 PM**

2 hrs. 17 mins before $t=0$ (6:00 PM)

2. Solve the initial value problem

$$(2xy + 3x^2)dx + (1 + 2y + x^2)dy = 0 \qquad y(1) = 2$$

M $\qquad$ N

- Check if it's exact

$$M_y = 2x \qquad N_x = 2x$$

equal

so it's exact! ✓

- $F_x = M = 2xy + 3x^2$

↓ integrate with respect to x

$$F = x^2y + x^3 + g(y)$$

↓ differentiate w/ respect to y

$$F_y = x^2 + g'(y) = 1 + 2y + x^2$$

$F_y = N = 1 + 2y + x^2$ ↓

$$g'(y) = 1 + 2y$$

$$g(y) = \int 1 + 2y \, dy = y + y^2 + C$$

$$F = x^2y + x^3 + y + y^2 + C$$

$$\boxed{x^2y + x^3 + y + y^2 = C}$$

general implicit solution

- plug in $y(1)=2$

$$\underbrace{1^2(2)+1^3+2+(2)^2}_{9}=C$$

$$\boxed{x^2y+x^3+y+y^2=9}$$

unique implicit solution

3. Find the particular solution to the Differential Equation $2y''+5y'+3y=0$ so that it satisfies $y(0)=3$ and $y'(0)=-4$.

- Find characteristic / auxillary eqn.

$$2r^2+5r^1+3r^0=0$$

$$2r^2+5r+3=0$$

- Find roots

$$r=\frac{-5\pm\sqrt{25-4(2)(3)}}{2(2)}=\frac{-5\pm\sqrt{1}}{4}$$

$$=-1 \text{ or } -\frac{3}{2}$$

2 distinct real roots

(1 repeated root
2 complex roots)

- General solution

$$y=C_1e^{-x}+C_2e^{-\frac{3}{2}x}$$

- Use initial conditions to find C_1 and C_2

$$y(0)=3=C_1e^0+C_2e^0$$

$$3=C_1+C_2 \quad \Delta$$

$$\rightarrow C_1=3-C_2$$

$y'(0) = -4$

$y' = -C_1 e^{-x} - \frac{3}{2} C_2 e^{-\frac{3}{2}x}$

$-4 = -C_1 e^{0} - \frac{3}{2} C_2 e^{0}$

$-4 = -C_1 - \frac{3}{2} C_2$ *

$3 = C_1 + C_2$

$+\left(-4 = -C_1 - \frac{3}{2} C_2\right)$

$-1 = -\frac{1}{2} C_2$

$2 = C_2$ ⟶ $C_1 = 3 - C_2$

$= 3 - 2$

$= 1$

$$y = 1 e^{-x} + 2 e^{-\frac{3}{2}x}$$

4. Find the particular solution to the Differential Equation $y'' - 2y' + 2y = 0$ so that it satisfies $y(0) = 1$ and $y'(0) = 2$.

- Find characteristic/ auxillary eqn.

$$r^2 - 2r + 2 = 0$$

- Find roots

$$r = \frac{2 \pm \sqrt{4 - 4(1)(2)}}{2(1)} = \frac{2 \pm \sqrt{(-1)4}}{2} = \frac{2 \pm 2i}{2} = 1 \pm 1i$$

α β

- Find General solution

$$y = e^{1x}\left(C_1 \cos(1x) + C_2 \sin(1x)\right)$$

$$= e^{x}\left(C_1 \cos(x) + C_2 \sin(x)\right)$$

- Use initial conditions to find C_1 and C_2

$$y(0) = 1 = \underbrace{e^0}_{1}\left(C_1 \underbrace{\cos(0)}_{1} + C_2 \cancel{\sin(0)}\right)$$

$$1 = C_1$$

$$y'(0) = 2$$

$$y' = e^x\left(C_1 \cos(x) + C_2 \sin(x)\right) + e^x\left(-C_1 \sin(x) + C_2 \cos(x)\right)$$

$$2 = e^0\left(C_1 \cos(0) + \cancel{C_2 \sin(0)}\right) + e^0\left(-C_1 \cancel{\sin(0)} + C_2 \cos(0)\right)$$

$$2 = C_1 + C_2 = 1 + C_2$$

$$1 = C_2$$

$$y = e^x(\cos(x) + \sin(x))$$

5. Find the general solution to $y''' + 2y'' + 5y' = 0$.

- Find characteristic/auxillary eqn.

$$r^3 + 2r^2 + 5r = 0$$

$$r(r^2 + 2r + 5) = 0$$

- Find roots

$r = 0$

$r = \frac{-2 \pm \sqrt{4 - 4(1)(5)}}{2(1)} = \frac{-2 \pm \sqrt{(-1)16}}{2} = -1 \pm 4i$

$\alpha \quad \beta$

- Find general solution

$$y = C_1 e^{0x} + e^{-1x}\left(C_2 \cos(4x) + C_3 \sin(4x)\right)$$

$$\boxed{y = C_1 + e^{-x}\left(C_2 \cos(4x) + C_3 \sin(4x)\right)}$$