MA 241: Exam 1

Practice exam

This exam is worth a total of **100 points**.

Directions: Write your full name and Test Form (A or B) on the front of the Blue Book. In the *Box No.* fill in your *Row Letter, Seat Number.*
All work must be shown in the Blue Book to receive credit, and only work in the Blue Book will be graded. Number all questions, including parts, and box your final answers.
No phones, notes, calculators, or other aids are allowed.

1. Calculate $\boxed{12}$
$$\int_1^2 x^3 \ln(x)\, dx$$

2. Find the arclength of the curve $y = \frac{1}{2}e^x + \frac{1}{2}e^{-x}$ from $x = 1$ to $x = 5$. $\boxed{13}$

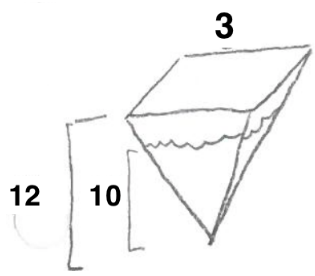
3. Find the average value of the function $f(x) = \frac{e^{\sqrt{x}}}{\sqrt{x}}$ on $[4, 9]$. $\boxed{12}$

4. A Force of 2lbs. is required to stretch a spring from it's natural length of $4ft.$ to a length of $8ft.$ How much work is done stretching the spring from $4ft.$ to $10ft.$? Your answer should include units. $\boxed{12}$

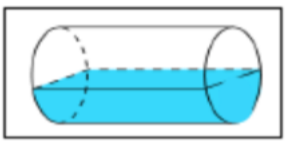
5. A 5m. chain with a weight of 30 N is lying on the ground. Find the work needed to raise one end to a height of 4m. Your answer should include units. $\boxed{12}$

6. A homogeneous lamina is enclosed by the graphs of $y = x^{\frac{3}{2}}$, $x = 0$, $x = 2$, $y = 0$. Find the coordinates for the center of mass. $\boxed{13}$

7. A gasoline storage tank in the shape of an inverted square-based pyramid (vertex at the bottom) has a height of $12m.$ and a base area of $9m.^2$. It is filled with gasoline to a height of $10m.$ Setup (DO NOT EVALUATE) an integral to find the work required to pump all of the water out the top of the tank. The mass density of gasoline is $720kg/m^3$ and gravity is $9.8m/s^2$. Your answer should include units and a picture specifying the location of your x and y-axes. $\boxed{13}$



8. Consider a horizontally oriented cylindrical tank with radius $3ft.$ and $7ft.$ length. Assuming the tank is full of syrup (weight density $1000lb./ft.^3$), setup (DO NOT EVALUATE) an integral to find the force on one end. Your answer should include units and a picture specifying the location of your x and y-axes. $\boxed{13}$



$u = \ln(x) \qquad dv = x^3 dx$

$du = \frac{1}{x} dx \qquad v = \frac{x^4}{4}$

1. $$\int_1^2 x^3 \ln(x)\, dx = \left[\ln(x)\frac{x^4}{4}\right]_1^2 - \int_1^2 \frac{x^4}{4}\frac{1}{x}\, dx$$

$$= \left[\ln(x)\frac{x^4}{4}\right]_1^2 - \int_1^2 \frac{x^3}{4}\, dx$$

$$= \left[\ln(x)\frac{x^4}{4}\right]_1^2 - \left[\frac{x^4}{16}\right]_1^2$$

$$= \left[\ln(x)\frac{x^4}{4} - \frac{x^4}{16}\right]_1^2$$

$$= \left(\ln(2)\frac{16}{4} - \frac{16}{16}\right) - \left(\underbrace{\cancel{\ln(1)}}_{0}\frac{1}{4} - \frac{1}{16}\right)$$

$$= \ln(2)\,4 - 1 + \frac{1}{16}$$

$$= \boxed{4\ln(2) - \frac{15}{16}}$$

(2) $y = \frac{1}{2}e^x + \frac{1}{2}e^{-x}$

$\frac{dy}{dx} = \frac{1}{2}e^x - \frac{1}{2}e^{-x}$

$$\mathcal{L} = \int_a^b \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$$

$$= \int_1^5 \sqrt{1 + \left(\frac{1}{2}e^x - \frac{1}{2}e^{-x}\right)^2}\, dx$$

$$= \int_1^5 \sqrt{1 + \frac{1}{4}e^{2x} + 2\left(\frac{1}{2}e^x\right)\left(-\frac{1}{2}e^{-x}\right) + \frac{1}{4}e^{-2x}}\, dx$$

$$= \int_1^5 \sqrt{1 + \frac{1}{4}e^{2x} - \frac{1}{2} + \frac{1}{4}e^{-2x}}\, dx$$

$$= \int_1^5 \sqrt{\frac{1}{4}e^{2x} + \frac{1}{2} + \frac{1}{4}e^{-2x}}\, dx$$

$$= \int_1^5 \sqrt{\left(\frac{1}{2}e^x\right)^2 + 2\left(\frac{1}{2}e^x\right)\left(\frac{1}{2}e^{-x}\right) + \left(\frac{1}{2}e^{-x}\right)^2}\, dx$$

perfect square

$$= \int_1^5 \sqrt{\left(\frac{1}{2}e^x + \frac{1}{2}e^{-x}\right)^2}\, dx$$

$$= \int_1^5 \frac{1}{2}e^x + \frac{1}{2}e^{-x}\, dx$$

$$= \left[\frac{1}{2}e^x - \frac{1}{2}e^{-x}\right]_1^5$$

$$= \boxed{\left(\frac{1}{2}e^5 - \frac{1}{2}e^{-5}\right) - \left(\frac{1}{2}e - \frac{1}{2}e^{-1}\right)}$$

3 $f_{avg} = \frac{1}{9-4} \int_4^9 \frac{e^{\sqrt{x}}}{\sqrt{x}}\, dx$

$$u = \sqrt{x} = x^{1/2}$$
$$du = \tfrac{1}{2} x^{-1/2} dx$$
$$2\, du = \tfrac{1}{\sqrt{x}}\, dx$$

when $x=9 \to u=\sqrt{9}=3$

$x=4 \to u=\sqrt{4}=2$

$$= \frac{1}{9-4} \int_2^3 e^u (2\, du)$$
$$= \frac{2}{5} \int_2^3 e^u\, du$$
$$= \frac{2}{5} \left[e^u\right]_2^3$$
$$= \boxed{\frac{2}{5}\left(e^3 - e^2\right)}$$

we will also accept:

$$f_{avg.} = \frac{1}{9-4} \int_4^9 \frac{e^{\sqrt{x}}}{\sqrt{x}}\, dx$$

$$u = \sqrt{x} = x^{1/2}$$
$$du = \tfrac{1}{2} x^{-1/2}\, dx$$
$$2\, du = \tfrac{1}{\sqrt{x}}\, dx$$

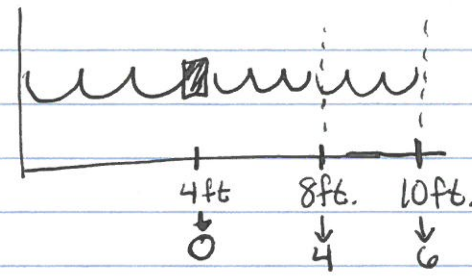
$$= \frac{2}{5} \int_{x=4}^{x=9} e^u\, du$$
$$= \frac{2}{5} \left[e^u\right]_{x=4}^{x=9}$$
$$= \frac{2}{5} \left[e^{\sqrt{x}}\right]_4^9$$
$$= \frac{2}{5}\left(e^{\sqrt{9}} - e^{\sqrt{4}}\right)$$
$$= \boxed{\frac{2}{5}\left(e^3 - e^2\right)}$$

(4) $f = Kx$

$2\text{lb.} = K(8\text{ft.} - 4\text{ft.})$

$= K(4\text{ ft.})$

$\frac{1}{2} = \frac{2}{4} = K$



so $W = \int_0^6 \frac{1}{2}x\,dx$

$= \left[\frac{x^2}{4}\right]_0^6$

$= \frac{36}{4} - 0$

$= \boxed{9 \text{ ft}\cdot\text{lbs}}$

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$x = 0\text{ m.}$ $\quad$ $x = 1\text{ m.}$ $\quad$ $x = 2\text{ m.}$

$f(0) = 0$

$f(1) = \left(\frac{30\text{ N}}{5\text{ m.}}\right)(1\text{ m.}) = 6\text{N}$

$f(2) = \left(\frac{30\text{ N}}{5\text{ m.}}\right)(2\text{ m.}) = 12\text{N}$

so $f(x) = \left(\frac{30\text{ N}}{5\text{ m}}\right)(x) = 6x$

$$W = \int_0^4 6x \, dx$$

$$= 3x^2 \Big]_0^4$$

$$= 3(16) - \cancel{3(0)}$$

$$= \boxed{48 \text{ J}}$$

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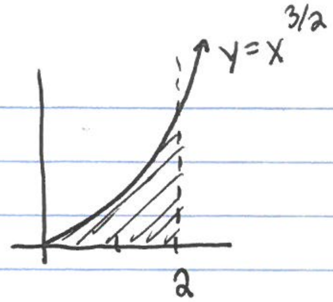

$$\int_0^2 x^{3/2}\,dx = \left[\frac{x^{5/2}}{5/2}\right]_0^2 = \left[\frac{2}{5}x^{5/2}\right]_0^2$$

$$= \frac{2}{5}\left(2^{5/2} - \cancel{0^{5/2}}\right)$$

$$= \boxed{\frac{2^{7/2}}{5}}$$

$$\int_0^2 x\, x^{3/2}\,dx = \int_0^2 x^{5/2}\,dx$$

$$= \left[\frac{x^{7/2}}{7/2}\right]_0^2 = \left[\frac{2}{7}x^{7/2}\right]_0^2$$

$$= \frac{2}{7}\left(2^{7/2} - \cancel{0^{7/2}}\right)$$

$$= \boxed{\frac{2^{9/2}}{7}}$$

$$\int_0^2 \frac{1}{2}\left(x^{3/2}\right)^2 dx = \int_0^2 \frac{1}{2}x^3$$

$$= \frac{1}{2}\left[\frac{x^4}{4}\right]_0^2 = \frac{1}{8}\left[x^4\right]_0^2$$

$$= \frac{1}{8}\left(2^4 - \cancel{0^4}\right)$$

$$= \boxed{2}$$

$$(\bar{x},\bar{y}) = \left(\frac{\left(\frac{2^{9/2}}{7}\right)}{\left(\frac{2^{7/2}}{5}\right)}, \frac{2}{\left(\frac{2^{7/2}}{5}\right)}\right)$$

$$= \left(\frac{2^{9/2}\cdot 5}{7\cdot 2^{7/2}}, \frac{2\cdot 5}{2^{7/2}}\right)$$

$$= \left(\frac{2\cdot 5}{7}, \frac{5}{2^{5/2}}\right)$$

$$= \boxed{\left(\frac{10}{7}, \frac{5}{2^{5/2}}\right)}$$

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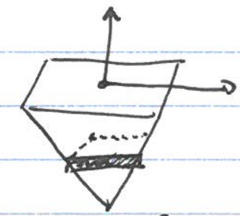
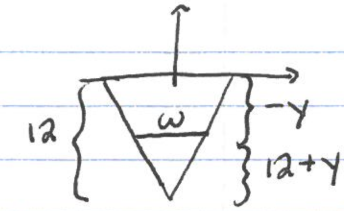
Option 1:





$A(y) = w^2$

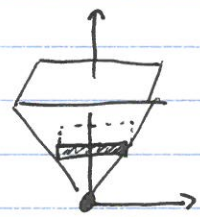
$\frac{w}{12+y} = \frac{3}{12}$

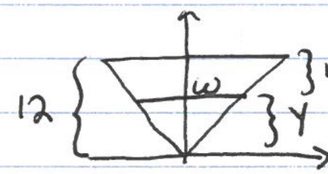
$w = \frac{1}{4}(12+y)$

$A(y) = \frac{1}{16}(12+y)^2$

$$\boxed{W = \int_{-12}^{-2} (720 \cdot 9.8) \frac{1}{16}(12+y)^2 (-y)\, dy}$$

Option 2:





$A(y) = w^2$

$\frac{w}{y} = \frac{3}{12}$

$w = \frac{1}{4} y$

$A(y) = \frac{1}{16} y^2$

$$\boxed{W = \int_{0}^{10} (720 \cdot 9.8) \frac{1}{16} y^2 (12-y)\, dy}$$

More options possible, just stay consistent with the coordinates you choose.

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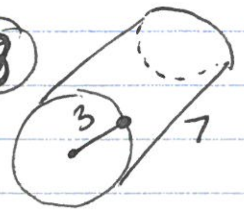
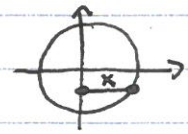
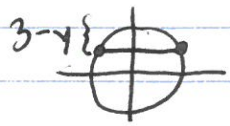


Option 1:



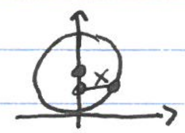
$$x^2+y^2=3^2$$
$$x=\sqrt{9-y^2}$$



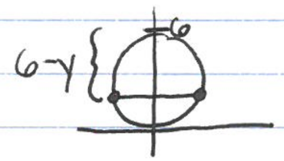
$$L(y)=2x$$
$$=2\sqrt{9-y^2}$$

$$\boxed{HF=\int_{-3}^{3}(100)\,2\sqrt{9-y^2}\,(3-y)\,dy}$$

Option 2:



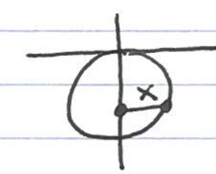
$$x^2+(y-3)^2=3^2$$
$$x=\sqrt{9-(y-3)^2}$$



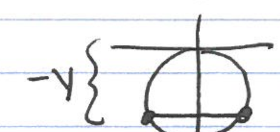
$$L(y)=2x$$
$$=2\sqrt{9-(y-3)^2}$$

$$\boxed{HF=\int_{0}^{6}(100)\,2\sqrt{9-(y-3)^2}\,(6-y)\,dy}$$

Option 3:



$$x^2+(y+3)^2=3^2$$
$$x=\sqrt{9-(y+3)^2}$$



$$L(y)=2x$$
$$=2\sqrt{9-(y+3)^2}$$

$$\boxed{HF=\int_{-6}^{0}(100)\,2\sqrt{9-(y+3)^2}\,(-y)\,dy}$$

More options are possible, just stay consistent with the coordinates you choose.