

This test is worth a total of **100 points**.

1. Answer the following as either **True or False**. If the answer is False, give a brief explanation **why**. 15

(a) If we used the Trapezoidal Rule with n trapezoids to approximate $\int_a^b f(x) dx$, then the exact error of our approximation will always equal $\frac{k(b-a)^3}{12n^2}$, where $|f''(x)| \leq k$ for $a \leq x \leq b$.

(b) The first step to decompose $\frac{x^3 - 6x^2 + 19x - 20}{(x-3)(x-1)}$ into a sum of partial fractions is to perform polynomial division.

(c) $\int_0^2 \frac{1}{x-1} dx = \ln|x-1| \Big|_0^2 = \ln|1| - \ln|-1| = 0$.

2. Evaluate $\int \frac{\sin^2(\ln(x)) \cos^5(\ln(x))}{x} dx$. 16

3. Use trigonometric substitution to evaluate $\int \frac{x^3}{\sqrt{9x^2 + 49}} dx$. 20

4. Evaluate $\int \frac{x-9}{x^3-2x^2+3x-6} dx$. 16

5. For this question, you may leave your answers in expanded form. (i.e. you do not need to plug it into a calculator)

(a) Approximate $\int_0^2 e^{3x} dx$ using Simpson's Rule with $n = 6$. 10

(b) Find the upper bound for the absolute value of the error. 9

Hint: $|E_S| \leq \frac{\tilde{k}(b-a)^5}{180n^4}$ where $|f^{(4)}(x)| \leq \tilde{k}$ for $a \leq x \leq b$.

6. Determine if the following integral is convergent or divergent. Evaluate the integral if it is convergent. Be sure to use proper mathematical notation. 14

$$\int_{1/3}^{\infty} \frac{9}{(1-3x)^4} dx$$

① a) False
 $\frac{K(b-a)^3}{12n^2}$ provides an interval that
 E_T is guaranteed to be in.

-OR-

$|E_T| \leq \frac{K(b-a)^3}{12n^2}$ so $|E_T|$ could be
less than $\frac{K(b-a)^3}{12n^2}$

b) True

we also
gave full
credit for

:"False, first you need to FOIL
then do polynomial division"

c) False

$\frac{1}{x-1}$ is not continuous when $x=1$,
so you can't use FTC here.

2

$$\int \frac{\sin^2(\ln(x)) \cos^5(\ln(x))}{x} dx$$

$$u = \ln(x)$$

$$du = \frac{1}{x} dx$$

+3
"u-sub"

$$\int \sin^2(u) \cos^5(u) du$$

$$\int \sin^2(u) \cos^4(u) (\cos u du)$$

$$\int \sin^2(u) (1 - \sin^2 u)^2 (\cos u du)$$

$$\tilde{u} = \sin(u)$$

$$d\tilde{u} = \cos(u) du$$

+8

Applying
"Case 2"

$$\int \tilde{u}^2 (1 - \tilde{u}^2)^2 d\tilde{u}$$

$$\int \tilde{u}^2 - 2\tilde{u}^4 + \tilde{u}^6 d\tilde{u}$$

← +2 "integrate"

$$\frac{1}{3} \tilde{u}^3 - \frac{2}{5} \tilde{u}^5 + \frac{1}{7} \tilde{u}^7 + C$$

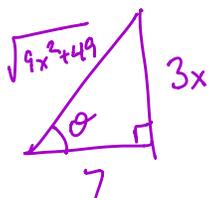
$$\frac{1}{3} \sin^3(u) - \frac{2}{5} \sin^5(u) + \frac{1}{7} \sin^7(u) + C$$

$$\frac{1}{3} \sin^3(\ln x) - \frac{2}{5} \sin^5(\ln x) + \frac{1}{7} \sin^7(\ln x) + C$$

+3 "substitute"

3

$$\int \frac{x^3}{\sqrt{9x^2+49}} dx$$



$$\begin{aligned} \tan \theta &= \frac{3x}{7} \\ \frac{7}{3} \tan \theta &= x \\ \frac{7}{3} \sec^2 \theta d\theta &= dx \\ \cos \theta &= \frac{7}{\sqrt{9x^2+49}} \\ \sqrt{9x^2+49} &= 7 \sec \theta \end{aligned}$$

+ 7
"performing the substitution"

$$\int \frac{(\frac{7}{3} \tan \theta)^3}{7 \sec \theta} \cdot \frac{1}{3} \sec^2 \theta d\theta$$

$$\int \frac{7^3}{3^4} \tan^3 \theta \sec \theta d\theta$$

$$\int \frac{7^3}{3^4} \tan^2 \theta (\tan \theta \sec \theta d\theta)$$

$$\int \frac{7^3}{3^4} (\sec^2 \theta - 1) (\tan \theta \sec \theta d\theta)$$

$$\begin{aligned} u &= \sec \theta \\ du &= \tan \theta \sec \theta d\theta \end{aligned}$$

+ 7
"Applying Case 1"

$$\int \frac{7^3}{3^4} (u^2 - 1) du$$

$$\frac{7^3}{3^4} \left(\frac{u^3}{3} - u \right) + C$$

← +2 "evaluate"

$$\frac{7^3}{3^4} \left(\frac{1}{3} \sec^3 \theta - \sec \theta \right) + C$$

$$\boxed{\frac{7^3}{3^4} \left(\frac{1}{3} \left(\frac{\sqrt{9x^2+49}}{7} \right)^3 - \frac{\sqrt{9x^2+49}}{7} \right) + C}$$

+4 "substitute"

Note: they can also use $\theta = \arctan(\frac{3x}{7})$ instead

$$\textcircled{4} \int \frac{x-9}{x^3-2x^2+3x-6} dx = \int \frac{x-9}{(x-2)(x^2+3)} dx = \int \frac{A}{x-2} + \frac{Bx+C}{x^2+3} dx$$

+5 "decomposing"

$$\begin{aligned} x-9 &= A(x^2+3) + (Bx+C)(x-2) \\ &= Ax^2 + 3A + Bx^2 - 2Bx + Cx - 2C \\ &= (A+B)x^2 + (-2B+C)x + (3A-2C) \end{aligned}$$

$$\begin{array}{l} A+B=0 \\ -2B+C=1 \\ 3A-2C=-9 \end{array} \left. \begin{array}{l} \rightarrow B=-A \\ \rightarrow -2(-A)+C=1 \\ \rightarrow +3A-2C=-9 \end{array} \right\} \begin{array}{l} -2(-A)+C=1 \\ 2A+C=1 \\ 4A+2C=2 \quad \downarrow \times 2 \\ \hline +3A-2C=-9 \\ \hline 7A = -7 \\ A = -1 \\ B = -(-1) = 1 \\ -2(1)+C=1 \\ C = 1+2 = 3 \end{array}$$

+4 "scratch work"

$$\int \frac{-1}{x-2} + \frac{1x+3}{x^2+3} = \int \frac{-1}{x-2} + \frac{x}{x^2+3} + \frac{3}{x^2+3} dx$$

$u = x^2+3$
 $du = 2x dx$

$$= -\ln|x-2| + \frac{1}{2} \ln|x^2+3| + \frac{3}{\sqrt{3}} \arctan\left(\frac{x}{\sqrt{3}}\right) + C$$

+7 "eval"

5

$$\Delta x = \frac{2-0}{6} = \frac{1}{3} \quad \left. \vphantom{\Delta x} \right\} +2$$

"coefficients" +3
"x-values" +2

$$a) S_6 = \frac{1}{3} \left[f(0) + 4f\left(\frac{1}{3}\right) + 2f\left(\frac{2}{3}\right) + 4f(1) + 2f\left(\frac{4}{3}\right) + 4f\left(\frac{5}{3}\right) + f(2) \right]$$

$$= \frac{1}{9} \left[e^{3(0)} + 4e^{3(1/3)} + 2e^{3(2/3)} + 4e^{3(1)} + 2e^{3(4/3)} + 4e^{3(5/3)} + e^{3(2)} \right]$$

+2
"evaluate"

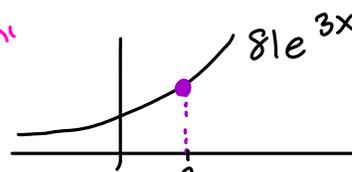
$$= \frac{1}{9} \left[e^0 + 4e^1 + 2e^2 + 4e^3 + 2e^4 + 4e^5 + e^6 \right]$$

$$= 134.8079...$$

Note: they can leave it expanded like this!

$$b) \begin{aligned} f(x) &= e^{3x} \\ f'(x) &= 3e^{3x} \\ f''(x) &= 9e^{3x} \\ f'''(x) &= 27e^{3x} \\ f^{(4)}(x) &= 81e^{3x} \end{aligned}$$

+4
"derivatives"



positive increasing
so max is when x=2

-OR- $f^{(5)}(x) = 243e^{3x}$
which is positive on $[0, 2]$
 $\Rightarrow f(x)$ is increasing
 \Rightarrow max is at $x=2$

+3
"maximize"

+2
"plug it in"

$$|E_5| \leq \frac{81e^6 (2-0)^5}{180 (6^4)} = \frac{81e^6 2^5}{180 6^4} = \frac{e^6}{90} = 4.4825...$$

6

$$\int_{1/3}^{\infty} \frac{9}{(1-3x)^4} dx$$

Caution vertical asymptote at $\frac{1}{3}$

$u = 1-3x$		when $x = \infty$
$du = -3 dx$		$u = 1-3(\infty) = -\infty$
$-\frac{1}{3} du = dx$		when $x = \frac{1}{3}$
		$u = 1-3(\frac{1}{3}) = 0$

+2
"u-sub"

$$\int_0^{-\infty} \frac{-3}{u^4} du$$

+2 "Breaking it up"

$$\int_{-1}^{-\infty} \frac{-3}{u^4} du$$

$$\int_0^{-1} \frac{-3}{u^4} du$$

$$\lim_{t \rightarrow -\infty} \int_{-1}^t \frac{-3}{u^4} du$$

+1
"notation"
+1
"integrate"

$$\lim_{t \rightarrow 0^-} \int_t^{-1} \frac{-3}{u^4} du$$

+1
+1

$$\lim_{t \rightarrow -\infty} \cancel{-3} \frac{u^{-3}}{\cancel{-3}} \Big|_{-1}^t$$

$$\lim_{t \rightarrow 0^-} \cancel{-3} \frac{u^{-3}}{\cancel{-3}} \Big|_t^{-1}$$

$$\lim_{t \rightarrow -\infty} \frac{1}{t^3} - \frac{1}{(-1)^3}$$

$$\lim_{t \rightarrow 0^-} \frac{1}{(-1)^3} - \frac{1}{t^3}$$

$$0 - \frac{1}{(-1)}$$

$$-1 - (-\infty)$$

$$= 1$$

$$= \infty$$

Thus, $\int_{1/3}^{\infty} \frac{9}{(1-3x)^4} dx$ diverges

+2 "conclusion"