NC STATE Math 241-003 - Study Guide - Fall 2021

Final Exam (with exercises for 4.7-4.9)

Be able to take all previous tests without error!
For more details, see each Chapter's Study Guide.

Chapter 0: Calc 1 Review

- Derivatives
- U-substitution
- Integration by Parts

Chapter 1: Applications of Integration

- Arclength
  Know the 3 formulas and when to apply them. Use perfect squares or u-sub to evaluate.
- Average Value
  Compute f_{avg}. Use Mean Value Theorem for Integration to find c so that $f(c) = f_{avg}$.
- Rope/Chain Pulling
  Find the amount of work done pulling a rope/chain from a to b.
- Springs
  Find the work done stretching a spring from a to b.
- Tank Pumping
  Setup the equation to find the amount of work done to pump liquid out the top of a tank.
- Hydrostatic Force
  Setup the equation to find the Hydrostatic force exerted on the flat side of a tank. OR Find the Hydrostatic force of a flat plate being submerged in water.
- Center of Mass
  Find the center of mass of a given region on the graph.

Chapter 2: Techniques of Integration

- Trig Integrals
  Solve integrals of the form $\int \sin^m(x)\cos^n(x)\,dx$ and $\int \tan^m(x)\sec^n(x)\,dx$.
- Trig Substitution
  Use Pythagorean Theorem and SOH CAH TOA to perform a θ substitution.
- Partial Fractions
  Decompose based on the 4 cases. Be able to find the constants and integrate.
- Numerical Integration
  Midpoint, Trapezoid, and Simpson's Rule. Find the error bound for the approximations.
- Improper Integrals
  Locate any asymptotes or discontinuities and use proper mathematical notation to evalutate the integral.

Chapter 3: Differential Equations

- Diff. Eq. Intro
  Know definition of Diff. Eq. and it's order. Verify if a given function is a solution to the Diff. Eq.
- Slope Fields
  Match a Diff. Eq. to it's slope field.
- Euler's Method
  Use the main formula to approximate a solution to an IVP.
- Separation of Variables
  Find the General solution to a separable Diff. Eq. Solve an Initial Value Problem.
- Applications
  Orthogonal Trajectories, Exponential Growth/Decay, Logistic Model, Newton's Heating & Cooling, Tank Mixing
- 2nd Order Linear Homogeneous
  Find the auxiliary equation. Use the 3 cases to find the general solution. Be able to solve an IVP
- 2nd Order Linear Non-Homogeneous
  Find the general solution.

Note: Springs & Circuits will NOT be on the exam.

Chapter 4: Sequences & Series

- Sequences
  Write a sequence in closed or recursive form. Use L'Hopital's and Squeeze Theorem to determine convergence/divergence.
- Intro to Series
  Write a series using summation notation. Know the main examples of series and their convergence/divergence results.
- Series Tests for Convergence/Divergence
  Divergence Test, Direct Comparison, Limit Comparison, Integral Test, Alternating Series, Absolute Convergence, & Ratio Test. Approximate series with the Integral and Alternating Series.
- Power Series
  Find interval and radius of convergence, and the center of the power series.
- Functions as Power series
  Write rational functions as power series. Take integrals/derivatives using power series. Identify interval and radius of convergence.
- Taylor & Maclaurin Series
  Derive the Taylor or Maclaurin series for functions. Manipulate the main examples of Maclaurin series to find a Maclaurin series.
- Taylor Polynomial
  Find the Taylor polynomial and estimate it's error.

EXERCISES FOR 4.7-4.9:
(Note: See the previous Chapters' Study Guides for review questions on the material you've been tested on already)

1. (a) Find the power series representation for $f(x) = \frac{5x}{1-3x^5}$
   (b) Use the series to evaluate $f'(x)$
2. (a) Find the power series representation for $g(x) = \frac{x^4}{9+x^2}$
   (b) Use the series to evaluate $\int g(x)\,dx$
3. (a) Derive a Maclaurin series for $\cos(x)$ (This means show all the work, no shortcuts!)
   (b) Find the Maclaurin series of $\cos(3x)$
   (c) Use the Maclaurin series to evaluate $\int \frac{1}{x^2}\cos(3x)\,dx$
4. Derive a Taylor series for $f(x) = \sin(x)$ centered at $a = \frac{\pi}{2}$
5. (a) Derive a Maclaurin series for e^x
   (b) Find the Maclaurin series of $e^{\sqrt{x}}$
   (c) Use the Maclaurin series to evaluate $\int e^{\sqrt{x}}\,dx$
6. Find the Taylor series for $f(x) = \frac{1}{x}$ centered at $a = 1$.
7. Use the Binomial series to expand the function as a power series. State the radius of convergence.
   (a) $(4+x)^{-1/3}$
   (b) $\frac{x}{\sqrt{1-x^2}}$
   (c) $\sqrt[4]{2-x}$
   (d) $\sqrt[5]{7+x}$
8. Find $T_2(x)$ for $f(x) = \sec(x)$ at $a = 0$.
9. (a) Find $T_4(x)$ for $f(x) = \frac{1}{\sqrt{x}}$ at $a = 1$.
   (b) Approximate $\frac{1}{\sqrt{2}}$ with $T_4(x)$
10. (a) Find $T_3(x)$ of $f(x) = e^x$ at $a = -2$.
    (b) For $-2.1 \le x \le -1.9$ estimate $|R_3|$.

Solutions:

1. (a) $\sum_{n=0}^{\infty} 5(3^n)x^{5n+1}$

   (b) $\sum_{n=0}^{\infty} 5(5n+1)(3^n)x^{5n}$

2. (a) $\sum_{n=0}^{\infty}(-1)^n\left(\frac{1}{9}\right)^{n+1}x^{2n+4}$

   (b) $\sum_{n=0}^{\infty}\frac{1}{2n+5}(-1)^n\left(\frac{1}{9}\right)^{n+1}x^{2n+5}+C$

3. (a) $\cos(x)=\sum_{n=0}^{\infty}\frac{(-1)^n x^{2n}}{(2n)!}$

   (b) $\cos(3x)=\sum_{n=0}^{\infty}\frac{(-1)^n 3^{2n}x^{2n}}{(2n)!}$

   (c) $\int \frac{1}{x^2}\cos(3x)\,dx=\sum_{n=0}^{\infty}\frac{(-1)^n 3^{2n}x^{2n-1}}{(2n)!(2n-1)}$

4. $\sin(x)=\sum_{n=0}^{\infty}\frac{(-1)^n}{(2n)!}(x-\frac{\pi}{2})^{2n}$

5. (a) $e^x=\sum_{n=0}^{\infty}\frac{x^n}{(n)!}$

   (b) $e^{\sqrt{x}}=\sum_{n=0}^{\infty}\frac{x^{n/2}}{(n)!}$

   (c) $\int e^{\sqrt{x}}\,dx=\sum_{n=0}^{\infty}\frac{x^{n/2+1}}{(n)!(\frac{n}{2}+1)}$

6. $\frac{1}{x}=\sum_{n=0}^{\infty}(x-1)^n(-1)^n$

7. (a) $4^{-1/3}\sum_{n=0}^{\infty}\binom{-\frac{1}{3}}{n}\left(\frac{x}{4}\right)^n$ and $R=4$.

   (b) $x\sum_{n=0}^{\infty}\binom{-\frac{1}{2}}{n}(-x^2)^n$ and $R=1$.

   (c) $2^{1/4}\sum_{n=0}^{\infty}\binom{\frac{1}{4}}{n}\left(-\frac{x}{2}\right)^n$ and $R=2$.

   (d) $7^{1/5}\sum_{n=0}^{\infty}\binom{\frac{1}{5}}{n}\left(\frac{x}{7}\right)^n$.

8. $T_2(x) = 1 + \frac{x^2}{2}$
9. (a) $T_4(x) = 1 - \frac{1}{2}(x-1) + \frac{3}{4(2!)}(x-1)^2 - \frac{15}{8(3!)}(x-1)^3 + \frac{(15)(7)}{16(4!)}(x-1)^4$
   (b) $\frac{1}{\sqrt{2}} \approx 1 - \frac{1}{2} + \frac{3}{4(2!)} - \frac{15}{8(3!)} + \frac{(15)(7)}{16(4!)}$
10. (a) $T_3(x) = e^{-2} + e^{-2}(x+2) + \frac{e^{-2}}{2!}(x+2)^2 + \frac{e^{-2}}{3!}(x+2)^3$
    (b) $|R_3| \leq \frac{e^{-1.9}}{4!}(0.1)^4$